

The taxonomic distribution of asteroids from multi-filter all-sky photometric surveys



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ABSTRACT

The distribution of asteroids across the main belt has been studied for decades to understand the current compositional distribution and what that tells us about the formation and evolution of our Solar System. All-sky surveys now provide orders of magnitude more data than targeted surveys. We present a method to bias-correct the asteroid population observed in the Sloan Digital Sky Survey (SDSS) according to size, distance, and albedo. We taxonomically classify this dataset consistent with the Bus and Binzel (Bus, S.J., Binzel, R.P. [2002]. *Icarus* 158, 146–177) and Bus–DeMeo et al. (DeMeo, F.E., Binzel, R.P., Slivan, S.M., Bus, S.J. [2009]. *Icarus* 202(July), 160–180) systems and present the resulting taxonomic distribution. The dataset includes asteroids as small as 5 km, a factor of three in diameter smaller than in previous work such as by Mothé-Diniz et al. (Mothé-Diniz, T., Carvano, J.M.Á., Lazzaro, D. [2003]. *Icarus* 162(March), 10–21). Because of the wide range of sizes in our sample, we present the distribution by number, surface area, volume, and mass whereas previous work was exclusively by number. While the distribution by number is a useful quantity and has been used for decades, these additional quantities provide new insights into the distribution of total material. We find evidence for D-types in the inner main belt where they are unexpected according to dynamical models of implantation of bodies from the outer Solar System into the inner Solar System during planetary migration (Levison, H.F., Bottke, W.F., Counelle, M., Morbidelli, A., Nesvorný, D., Tsiganis, K. [2009]. *Nature* 460(July), 364–366). We find no evidence of S-types or other unexpected classes among Trojans and Hildas, albeit a bias favoring such a detection. Finally, we estimate for the first time the total amount of material of each class in the inner Solar System. The main belt's most massive classes are C, B, P, V and S in decreasing order. Excluding the four most massive asteroids, (1) Ceres, (2) Pallas, (4) Vesta and (10) Hygiea that heavily skew the values, primitive material (C-, P-types) account for more than half main-belt and Trojan asteroids by mass, most of the remaining mass being in the S-types. All the other classes are minor contributors to the material between Mars and Jupiter.

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1. Introduction

The current compositional makeup and distribution of bodies in the asteroid belt is both a remnant of our early Solar System's primordial composition and temperature gradient and its subsequent physical and dynamical evolution. The distribution of material of different compositions has been studied based on photometric color and spectroscopic studies of ~2,000 bodies in visible and near-infrared wavelengths (Chapman et al., 1971, 1975; Gradie and Tedesco, 1982; Gradie et al., 1989; Bus, 1999; Bus and Binzel, 2002a; Mothé-Diniz et al., 2003). These data were based on all available spectral data at the time the work was performed including spectral

surveys such as Tholen (1984), Zellner et al. (1985), Barucci et al. (1987), Xu et al. (1995), Bus and Binzel (2002a), Lazzaro et al. (2004).

The first in-depth study showing the significance of global trends across the belt looked at surface reflectivity (albedo) and spectrometric measurements of 110 asteroids. It was then that the dominant trend in the belt was found: S-types are more abundant in the part of the belt closer to the Sun and the C-types further out (Chapman et al., 1975). Later work by Gradie and Tedesco (1982) and Gradie et al. (1989) revealed clear trends for each of the major classes of asteroids, concluding that each group formed close to its current location.

The Small Main-belt Asteroid Spectroscopic Survey (SMASII, Bus and Binzel, 2002b) measured visible spectra for 1447 asteroids and the Small Solar System Objects Spectroscopic Survey (S3OS2) observed 820 asteroids (Lazzaro et al., 2004). The conclusion of

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these major spectral surveys brought new discoveries and views of the main belt. Bus and Binzel (2002b) found the distribution to be largely consistent with Gradie and Tedesco (1982), however they noted more finer detail within the S and C complex distributions, particularly a secondary peak for C-types at 2.6 AU and for S-types at 2.85 AU. Mothé-Diniz et al. (2003) combined data from multiple spectral surveys looking at over 2000 asteroids with H magnitudes smaller than 13 ($D \sim 15$ km for the lowest albedo objects). Their work differed from early surveys finding that S-types continued to be abundant at further distances, particularly at the smaller size range covered in their work rather than the steep dropoff other surveys noted.

Only in the past decade have large surveys at visible and mid-infrared wavelengths been available allowing us to tap into the compositional detail of the million or so asteroids greater than 1 km that are expected to exist in the belt (Bottke et al., 2005). The results of these surveys (including discovery surveys), however, are heavily biased toward the closest, largest, and brightest of asteroids. This distorts our overall picture of the belt and affects subsequent interpretation.

In this work we focus on the data from the Sloan Digital Sky Survey Moving Object Catalog (SDSS, MOC, Ivezić et al., 2001, 2002) that observed over 100,000 unique asteroids in five photometric bands over visible wavelengths. These bands provide enough information to broadly classify these objects taxonomically (e.g., Carvano et al., 2011). In this work we refer to the SDSS MOC as SDSS for simplicity. We classify the SDSS data and determine the distribution of asteroids in the main belt. We present a method to correct for the survey's bias against the dimmest, furthest bodies.

Traditionally, the asteroid compositional distribution has been shown as the number objects of each taxonomic type as function of distance. While the number distribution is important for size-frequency distributions and understanding the collisional environment in the asteroid belt, the concern with this method is that objects of very different sizes are weighted equally. For example, objects with diameters ranging from 15 km to greater than 500 km were assigned equal importance in previous works. This is particularly troublesome for SDSS and other large surveys because the distribution by number further misrepresents the amount of material of each class by equally weighting objects that differ by two orders of magnitude in diameter and by six orders of magnitude in volume. To create a more realistic and comprehensive view of the asteroid belt we provide the taxonomic distribution according to number, surface area, volume, and mass. New challenges are presented when attempting to create these distributions including the inability to account for the smallest objects (below the efficiency limit of SDSS), the incompleteness of SDSS even at size ranges where the survey is efficient, and incomplete knowledge of the exact diameters, albedos and densities of each object. We attempt to correct for as many of these issues as possible in the present study.

The distribution according to surface area is perhaps the most technically correct result because only the surfaces of these bodies are measured. We only have indirect information about asteroid interiors, mainly derived from the comparison of their bulk density with that of their surface material, suggesting differentiation in some cases, and presence of voids in others (e.g., Consolmagno et al., 2008; Carry, 2012). The homogeneity in surface reflectance and albedo of asteroids pertaining to dynamical families (e.g., Ivezić et al., 2002; Cellino et al., 2002; Parker et al., 2008; Carruba et al., 2013) however suggest that most asteroids have an interior composition similar to their surface composition. Nevertheless, recent models find that large bodies even though masked with fairly primitive surfaces could actually have differentiated interiors (Elkins-Tanton et al., 2011; Weiss et al.,

2012). The distribution of surface area is relevant for dust creation from non-catastrophic collisions (e.g. Nesvorný et al., 2006, 2008) and from a resource standpoint such as for mining materials on asteroid surfaces. The volume of material provides context for the total amount of material in the asteroid belt with surfaces of a given taxonomic class. While we do not know the actual composition or properties of the interiors we can at least account for the material that exists.

The most ideal case is to determine the distribution of mass. This view accounts for all of the material in the belt, corrects for composition and porosity of the interior and properly weights the relative importance of each asteroid according to size and density. While the field is a long way away from having perfectly detailed shape and density measurements for every asteroid, by applying estimated sizes and average densities per taxonomic class to a large, statistical sample, we provide in this work the first look at the distribution of classes in the asteroid belt according to mass, and estimate the total amount of material each class represents in the inner Solar System.

Section 2 introduces the data used for this work. We overview observing biases and our correction method in Sections 3 and 4. We describe our classification method for our sample in Section 5. We then explain in Section 6 our method for building the compositional distribution and application of our dataset to all asteroids in the main belt. Finally, we present in Section 7 the bias-corrected taxonomic distribution of asteroid material across the main belt according to number, surface area, volume, and mass, and discuss the results in Section 8.

2. The dataset

2.1. Selection of high quality measurements from SDSS

The Sloan Digital Sky Survey (SDSS) is an imaging **and spectroscopy** survey dedicated to observing galaxies and quasars (Ivezić et al., 2001). The images are taken in five filters, u' , g' , r' , i' , and z' , from 0.3 to 1.0 μm . The survey also observed over 400,000 moving objects in our Solar System of which over 100,000 are unique objects linked to known asteroids. The current release of the Moving Object Catalogue (SDSS MOC 4, Ivezić et al., 2002) includes observations through March 2007.

We restrict our sample from the SDSS MOC database according to the following criteria. First, we keep only objects assigned a number or a provisional designation, i.e., those for which we can retrieve the orbital elements. We then remove observations that are deemed unreliable: with any apparent magnitudes greater than 22.0, 22.2, 22.2, 21.3, 20.5 for each filter (5.9% of the SDSS MOC4), which are the limiting magnitudes for 95% completeness (Ivezić et al., 2001), or any photometric uncertainty greater than 0.05 (excluding the u' filter, explained below). These constraints remove a very large portion of the SDSS dataset (about 87% of all observations), largely due to the greater typical error for the z' filter. While there is only a small subset of the sample remaining (Fig. 1), we are assured of the quality of the data. Additionally, for higher errors, the ambiguity among taxonomic classes possible for an object becomes so great that any classification becomes essentially meaningless. We exclude the u' filter from this work primarily because of the significantly higher errors in this filter compared to the others (Fig. 2), and secondarily because neither the Bus nor Bus-DeMeo taxonomies (that we use as reference for classification consistency, Bus and Binzel, 2002a; DeMeo et al., 2009) covered that wavelength range.

The fourth release of the MOC contains non-photometric nights in the dataset. The SDSS provides data checks that indicate

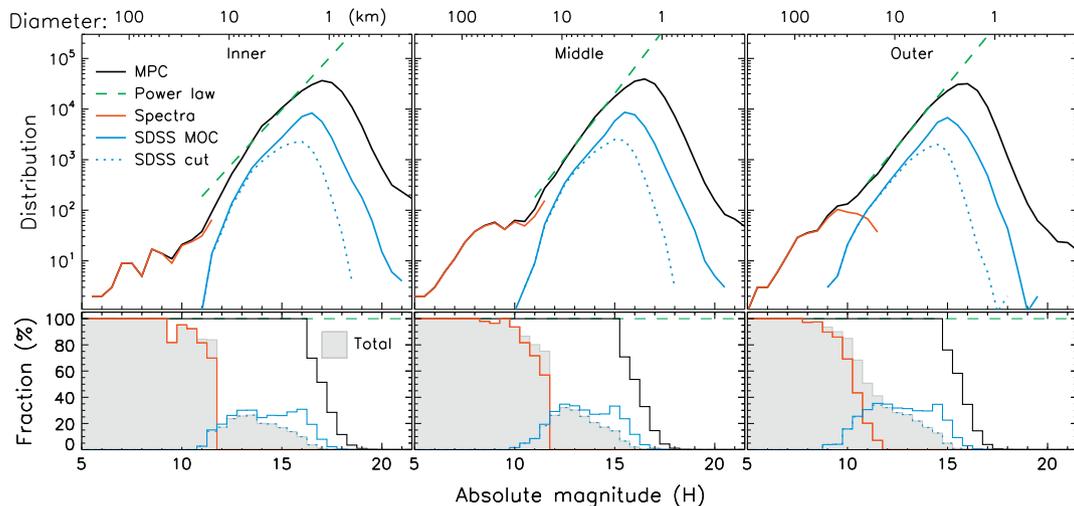


Fig. 1. Number (top) and fraction (bottom) of all asteroids discovered (solid black) and observed by spectroscopic surveys (red), or SDSS MOC (blue) in each zone of the main belt. The subset of SDSS MOC we used here (with cuts applied to photometry, see Section 2.1) is shown in dotted blue. The completeness of discovered asteroids at each size range is determined by extrapolating the expected population using a power law fit (dashed green) to the MPC list of discovered asteroids (solid black). The power law indices calculated in this work (see Section 6.3) for the IMB, MMB, and OMB (determined over the H magnitude range 14–16, 13–15, and 12–14.5) are -2.15 , -2.57 , and -2.42 , respectively. In the bottom panel the total fraction of the sample (before bias correction) is shaded in gray. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

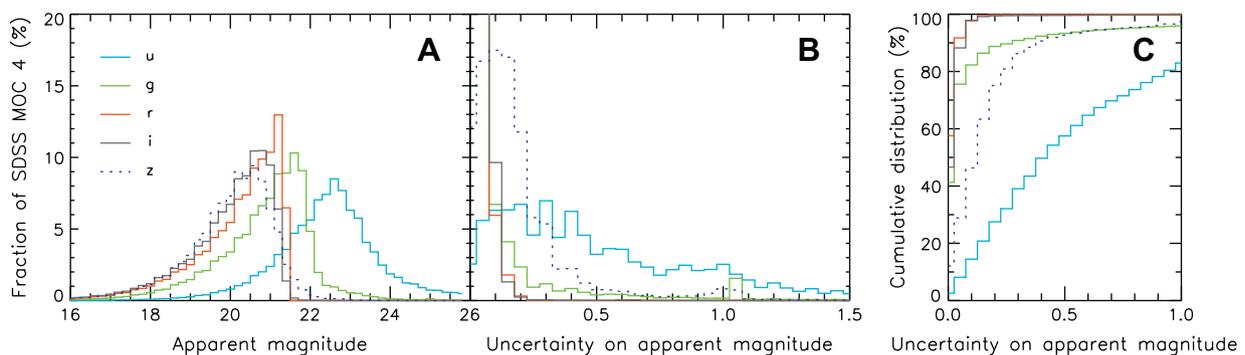


Fig. 2. Distribution of the apparent magnitude (A) and associated uncertainty (B) for all the SDSS MOC4 observations (471,569). The larger uncertainty affecting the observations in the u filter (C) precludes any reliable classification information to be retrieved from this filter.

potential problems with the measurements,¹ and we thus remove observations with flags relevant to moving objects and good photometry: `edge`, `badsky`, `peaks too close`, `not checked`, `binned4`, `nodeblend`, `deblend degenerate`, `bad moving fit`, `too few good detections`, and `stationary`. These flags note issues such as data where objects were too close to the edge of the frame, the peaks from two objects were too close to be deblended, the object was detected only in a 4×4 binned frame, or the object was not detected as moving. Further details of the flags are provided on the websites in the footnotes. The presence of these flags does not necessarily imply problematic data, but because the observations removed due to these flags represent a small percentage of the total objects that fall within the magnitude and photometric error constraints ($\sim 2\%$), we prefer to slightly restrict the sample than to contaminate it. Of the 471,569 observations in MOC4 we have a sample of 58,607 observations after applying the selection criteria. We keep observations that are flagged as having interpolation (37% of our sample), including `psf flux interp` (26% of our sample) which indicates that over

20% of the point spread function flux is interpolated. We also include observations corrected for cosmic rays (6.5%) and those that might have a cosmic ray but are uncorrected (1.5%). Anyone wishing to use the SDSS data or classification results to analyze particular objects rather than large populations is cautioned to note all flags associated with an observation.

2.2. Average albedo of each taxonomic class

There have been recent efforts to determine average albedos per taxonomic class (Ryan et al., 2010; Usui et al., 2011; Masiero et al., 2011). These results can be used to more accurately estimate the diameter of a body of a given taxonomic class. In some cases, however, the results disagree by more than the reported uncertainties (e.g., B-types, see Table 1). We calculate mean values, weighted by the number of albedos determined and their accuracy, for each taxonomic class for this work based on averages reported from previously published results (Ryan et al., 2010; Usui et al., 2011; Masiero et al., 2011). See Table 1 for a summary of published values and the averages we use in this work. It must also be noted that the average albedo per class does not necessarily represent the actual albedo for any particular

¹ <http://www.sdss.org/dr4/products/catalogs/flags.html>, http://www.sdss.org/dr4/products/catalogs/flags_detail.html, <http://www.sdss.org/dr7/tutorials/flags/index.html>, http://www.astro.washington.edu/users/ivezic/sdssmoc/moving_flags.txt.

Table 1
Summary of albedo determination for asteroids listed in DeMeo et al. (2009) based on radiometry, using data from IRAS (Ryan et al., 2010), AKARI (Usui et al., 2011), and WISE (Masiero et al., 2011) infrared satellites. The two columns for IRAS correspond to two different thermal models applied to the data set (STM and NEATM, see Ryan et al., 2010, for details). The mean albedo (last column) is obtained by averaging all the determinations, weighted by their accuracy and number.

Class	IRAS			AKARI		WISE		Average a
	#	a_{STM}	a_{NEATM}	#	a	#	a	
A	4	0.26 ± 0.12	0.18 ± 0.04	6	0.23 ± 0.06	5	0.19 ± 0.03	0.20 ± 0.03
B	2	0.26 ± 0.13	0.08 ± 0.09	3	0.14 ± 0.03	2	0.12 ± 0.02	0.14 ± 0.04
C	42	0.08 ± 0.02	0.06 ± 0.01	44	0.06 ± 0.03	32	0.06 ± 0.03	0.06 ± 0.01
D	11	0.08 ± 0.03	0.07 ± 0.03	14	0.06 ± 0.03	13	0.05 ± 0.03	0.06 ± 0.01
K	12	0.16 ± 0.07	0.12 ± 0.04	14	0.14 ± 0.04	11	0.13 ± 0.06	0.14 ± 0.02
L	12	0.14 ± 0.04	0.11 ± 0.04	16	0.12 ± 0.04	19	0.15 ± 0.07	0.13 ± 0.01
Q	1	0.51 ± 0.10	0.41 ± 0.08	1	0.28 ± 0.01	1	0.15 ± 0.03	0.27 ± 0.08
S	50	0.26 ± 0.06	0.20 ± 0.06	104	0.23 ± 0.05	121	0.22 ± 0.07	0.23 ± 0.02
V	1	0.37 ± 0.08	0.35 ± 0.07	1	0.34 ± 0.01	8	0.36 ± 0.10	0.35 ± 0.01

object because albedo may vary greatly among each class (e.g., Masiero et al., 2011).

The X class is divided into three classes, E, M, and P, distinguished solely by their albedo ($P < 0.075$, $0.075 < M < 0.30$, $E > 0.30$). We calculate average albedo in each class from the roughly 2,000 objects in our sample that have WISE, AKARI, or IRAS albedos. We find average albedos of 0.45, 0.14, and 0.05 for E, M, and P, respectively. Because the average albedo for a given class is calculated solely using objects with spectral data, and the spectral measurements are biased toward brighter, higher albedo objects, this average could consequently be biased toward higher albedos.

2.3. Average density of each taxonomic class

To convert from number of objects to mass, the average density for each class is crucial. Recently, an order of magnitude improvement of the sample of asteroid density estimates to 287 allowed the computation of the average density for each taxonomic class (Carry, 2012). In that work, multiple average densities are reported depending on the cutoff quality of measurements included. For the densities used in this work we chose the average densities using only the highest quality measurements (despite the smaller sample size). While these values are certainly an improvement over assuming the same density for all asteroids, there is still significant uncertainty in the real densities for any single object, and there is likely a correlation between density and size particularly due to differences in macroporosity (see Carry (2012) for details). Because we use such a large sample, the differences between any single asteroid and the average should have only a minor effect on the outcome.

For E, M, and P class objects no average density was reported in Carry (2012). In this work we take all objects with densities in each of those classes and calculate average densities for each class. We find densities of $\rho_E = 2.8 \pm 1.2$, $\rho_M = 3.5 \pm 1.0$, and $\rho_P = 2.7 \pm 1.6$ g/cm³. The density of M-types is the highest which is consistent with the current interpretation for their composition. Some objects in that class are thought to be metallic, and to contain significant amounts of dense iron (e.g., Gaffey et al., 1989; Lipschutz et al., 1989; Bell et al., 1989). However, the M-class is degenerate in both visible spectrum and geometric albedo because multiple kinds of asteroids are known to fall in that category each having different composition and density (see Rivkin et al., 2000; Shepard et al., 2008; Ockert-Bell et al., 2010 among others.) Not enough data are available to confidently distinguish the distributions of the different objects falling in the M-class so we group them together in this work. Additionally, as no density measurements are available for the D class, we assign an average density of 1 g/cm³, a density consistent with comets and transneptunian objects from the outer Solar System (Carry, 2012).

3. Observing biases

Asteroid observations over visible wavelengths are subject to multiple biases, and the SDSS dataset is no exception. Detection biases for automatic surveys (relevant to discovery surveys as well as SDSS) are due to properties of the asteroid (such as size, albedo, and distance), the physical equipment (such as telescope size and CCD quality), the scan pattern of the sky, and the software's automatic detection algorithm. For a thorough description of asteroid observing biases see Jedicke et al., 2002.

Efforts to correct the observed asteroid distribution from observing biases have been undertaken for decades (Chapman et al., 1971; Kiang, 1971; Gradie and Tedesco, 1982; Gradie et al., 1989; Bus, 1999; Stuart et al., 2004). One of the most significant is a bias toward observing objects with the highest apparent brightness (objects that are larger, closer, or have a higher surface albedo). This bias is particularly important for the smallest asteroids, where the incompleteness of observed versus as-yet undiscovered asteroids is considerable for any magnitude-limited survey.

The basis of relating the information in the given dataset to the entire suite of asteroids done here is fundamentally the same as in most previous work, however, it is executed slightly differently. In previous work (e.g., Kiang, 1971; Chapman et al., 1971, 1975; Gradie and Tedesco, 1982; Zellner et al., 1985; Bus, 1999; Bus and Binzel, 2002b; Mothé-Diniz et al., 2003) the asteroid belt is broken up into bins based on orbital elements (typically semi-major axis but some works include inclination as well) and brightness (earlier works used the apparent magnitude in V but later works used the absolute magnitude H). A correction factor is calculated as the total discovered numbered objects in each bin divided by the total number of objects in each bin in the given dataset. Each object in the dataset is then multiplied by the appropriate correction factor.

In this work we determine the fraction of each taxonomic class in each bin from our dataset and apply those fractions to the total number of discovered objects. These methods are most accurate if the original dataset is essentially an unbiased dataset and assume the relative fractions in each bin in the given dataset represent the actual relative fractions of all asteroids. We describe in this work many steps to both minimize bias in the dataset and to most accurately compare objects of similar size. These include using the average albedo per taxonomic class to move from an H magnitude-limited to diameter -limited sample and correcting for discovery incompleteness at large H magnitudes. This work accounts for both the sensitivity difference between the inner and outer parts of the belt and uses a dataset sensitive enough to probe to very small sizes.

Many of the previous spectroscopic surveys were subject to a target selection bias. These surveys focused more heavily on objects within asteroid families making the sample weighted more

strongly toward these objects. Previous work included a correction for these biases (e.g., Bus and Binzel, 2002a; Mothé-Diniz et al., 2003). However, because the SDSS is an **automated** survey that does not specifically target any type of objects or region of the belt it does not have the bias of many of the asteroid spectroscopic surveys that targeted specific regions.

It is also arguable that, even after correcting for this selection bias, counting family members overweights the importance of the original parent body in terms of overall compositional distribution. Even with an ideal, unbiased dataset, if one counts each asteroid with an equal weight (for example, by number) the compositional distribution will be heavily weighted toward the asteroid families even though all the family members are essentially of the same composition and originate from the same body. This is fine for studies of number distributions, but not for the distribution of total material. A way to mitigate this oversampling of families is to explore the distribution in terms of volume or mass as explained in the introduction. In this case we are counting all contributed material of the family; in essence we are putting the ejected fragments back together again and accounting for the total amount of material.

Accounting for the bias amongst the smallest asteroids is common to many datasets. Unique to SDSS compared to previous spectroscopic work is the bias against observing the largest, brightest asteroids because they saturated the SDSS detector. Any study of the SDSS sample would need to correct for the missing large asteroids.

4. Defining the least-biased subset

4.1. Corrections for the largest, brightest asteroids

SDSS did not have the capability to measure the largest, brightest asteroids. Conveniently, past spectroscopic surveys are nearly complete at these sizes and fill in that gap (Fig. 1).

We include the taxonomic classes for 1488 asteroids with an absolute magnitude $H < 12$ determined using spectroscopic measurements in the visible wavelengths (Zellner et al., 1985; Bus and Binzel, 2002b; Lazzaro et al., 2004; DeMeo et al., 2009), available on the Planetary Data System (Neese, 2010). We keep only the large objects from these surveys where spectroscopic sampling is nearly complete (>90%). The smaller objects in the spectroscopic surveys ($H > 12$) were not included in this work because they are more subject to observing biases and selection criteria (Mothé-Diniz et al., 2003). If an object was observed both in the spectroscopic surveys and the SDSS dataset, we use the data and classification from the spectroscopic surveys.

4.2. Corrections for the smallest, dimmest asteroids

Rather than extrapolating into regions in which we have no data that could misrepresent reality, we instead remove the biased portions of the data. We determine the size of the smallest, dark asteroid at a far distance (in this case, the outer belt) at which the SDSS survey is highly efficient. This number is based on the magnitude limits given by Ivezić et al. (2001) and the turnover in objects detected in the survey as a function of size (described in the next paragraph). We then remove any asteroids from the sample that are smaller than that limit. In essence, we create a sample restricted by a physical rather than an observable quantity: a diameter-limited instead of an apparent magnitude-limited sample.

Taking the SDSS sample, we determine the largest absolute magnitude (H) at which the survey is sensitive for each zone. We present in Fig. 1 the number of objects and fraction of the sample covered by the spectroscopic surveys as well as the fraction the

SDSS covers relative to all discovered and undiscovered asteroids for a large range of absolute magnitudes. The peak of the black solid line in Fig. 1 represents the limit of discovery efficiency for zones of the main belt. The cutoff magnitudes are roughly 17.2, 16.5, 15.5, 14.5, and 12.5 for the inner (IMB), middle (MMB), and outer main belt (OMB), Cybeles and Hildas, and Trojans, respectively.

We use these absolute magnitude limits to define the asteroid size range for which a distribution study can be reasonably confident. The smallest size sampled among all asteroid types is limited by the darkest, farthest objects (P-type, see Table 2). For our sample we use the outer main belt to determine our size cutoff. It would be preferable to use the Hilda or Trojan regions, because then we explore the same size range from the main belt out to the Trojans. However, this would drastically limit our sample size. It is thus important to recognize that our results do not contain Hildas and Trojans down to as small sizes as in the main belt. In our sample, the number of Hildas and Trojans is severely biased toward larger sizes, however, because these populations contain asteroids all with similarly low albedos (Grav et al., 2011, 2012a,b) there is no significant bias on the relative number of bodies of each taxonomic class. For this reason we include the Hildas and Trojans in the present work.

The smallest P-type asteroids the SDSS surveyed in the OMB have $H = 15.5$ which represents a diameter of ~ 5 km. While we sample, for example, S-types in the outer belt and C-types in the inner belt with diameters of ~ 2 km and S- and V-types in the inner belt to 1 km or less, including these smaller objects in our sample would bias the results in terms of number toward these smaller objects that are not sampled in the outer belt. Instead we include in our sample only objects that are 5 km or larger. This size is equivalent to a different H magnitude for each class. The ratio of each taxonomic class' albedo (a_i , where i is the taxonomic class) with the P-type albedo (a_p) can be used to determine the magnitude difference between same-size objects of different taxonomic classes using the equation

$$H_i - H_p = 2.5 \log \frac{a_p}{a_i} \quad (1)$$

We cut the sample of each taxonomic class according to these H magnitude limits, which are listed in Table 2. The average albedo for each class was determined by taking the average of the albedo determined for each class from IRAS, AKARI, and WISE (Ryan et al., 2010; Usui et al., 2011; Masiero et al., 2011, see Section 2.2). Using a different H magnitude for each taxonomic class is critical. If we cut our sample at $H = 15.5$ for all objects we would be comparing, for example, 5 km P-types to 2 km S-types, which are much more numerous owing to the steep size–frequency distribution of the asteroid population.

Table 2

Cuts on the absolute magnitude for each taxonomic class. These cutoffs were determined by the limiting case of P-type asteroids in the outer belt. Average density (in g/cm^3 , from Carry, 2012) and albedo (see Section 2.2) are also reported. We choose a density of D-types of 1 g/cm^3 , consistent with an outer Solar System origin because no D-type densities have been accurately measured.

Class	H_{cut}	Density	Albedo
A	13.99	3.73 ± 1.40	0.20 ± 0.03
B	14.38	2.38 ± 0.45	0.14 ± 0.04
C	15.30	1.33 ± 0.58	0.06 ± 0.01
D	15.30	1.00 ± 1.00	0.06 ± 0.01
K	14.38	3.54 ± 0.21	0.14 ± 0.02
L	14.46	3.22 ± 0.97	0.13 ± 0.01
S	13.84	2.72 ± 0.54	0.23 ± 0.02
V	13.39	1.93 ± 1.07	0.35 ± 0.01
E	13.12	2.67 ± 1.20	0.45 ± 0.21
M	14.49	3.49 ± 1.00	0.13 ± 0.05
P	15.50	2.84 ± 1.60	0.05 ± 0.01

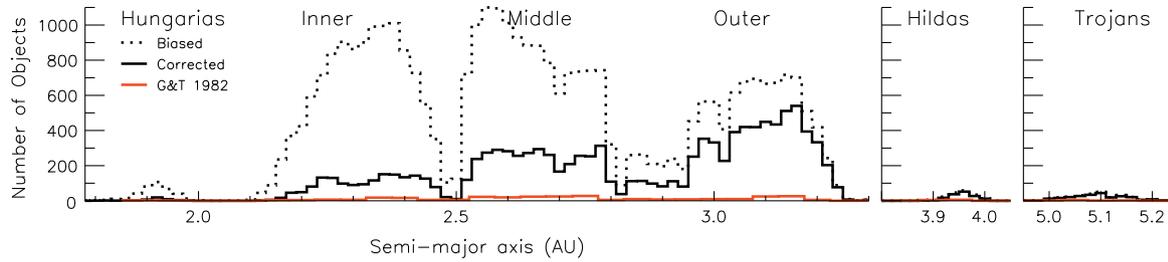


Fig. 3. Number of asteroids as a function of heliocentric distance for three different samples: our original sample made of spectroscopic surveys and SDSS photometry (34,503 asteroids, dashed line), our bias-corrected sample (13,211 solid line), and the sample of 656 taxonomically classified asteroids from Gradie and Tedesco (1982).

The size of the SDSS sample before and after the bias-correction selection is shown in Fig. 3., together with the number of objects presented in the preceding work by Gradie and Tedesco (1982). It is clear that a vast number of objects are removed from the inner and middle sections of the belt because they are below the critical size limit. To give an estimate on the importance of this size correction, there are ~ 5000 5 km asteroids in the middle belt, however there are about $\sim 40,000$ 2 km ones known, nearly a factor of 10 greater.

5. Taxonomic classification

The SDSS asteroid data has been grouped and classified according to their colors by many authors. Ivezić et al. (2002) classified the C, S, and V groups using the $z'-i'$ color and the first principal component of the $r'-i'$ versus $r'-g'$ colors. Nesvorný et al. (2005) used the first two principal components of u' , g' , r' , i' , z' colors and distinguished between the C, X, and S-complexes. Carvano et al. (2011) converted colors to reflectance values and created a probability density map of previously classified asteroids and synthetic spectra to classify the SDSS dataset.

In this work we seek to maximize the taxonomic detail contained in the dataset and strive to keep the class definitions as consistent as possible with previous spectral taxonomies that were based on higher spectral resolution and larger wavelength coverage data sets, specifically Bus (Bus and Binzel, 2002a) and Bus–DeMeo (DeMeo et al., 2009) taxonomies.

5.1. Motivation for manually defined class boundaries

The best way to mine the most information out of such a large dataset could be to perform an analysis of the variation and clustering. Methods such as Principal Component Analysis or Hierarchical Clustering could separate and highlight groups within the data. The advantage to automated methods is they are unbiased by human intervention and can efficiently characterize large datasets, which are the motivations for many unsupervised classifications.

However, because most of our understanding of asteroid mineralogy comes from relating asteroid spectral taxonomic classes to meteorite classes and comparing absorption bands, we find it more relevant to connect this low-resolution data to already defined and well-studied asteroid taxonomic classes (that were based on Principal Component Analysis). This facilitates putting the SDSS results in context with the findings from other observations that have accumulated over decades. To classify the data we started with the class centers and standard deviations (based on data used to create the Bus–DeMeo taxonomy converted to SDSS colors) to calculate the distance of each object to the class center.

Considering the above, while we still use the class centers and calculated deviations as a guide, we choose to fix boundaries for each class and manually tweak them (as described below) according to the data to best capture the essence of each class. A negative

consequence of fixed boundaries is that near the boundary objects exist on either side that may have very similar characteristics though are classified differently (as opposed to methods which assign a probability for each object to be in a certain class). Additionally, a human bias is added. The advantage, however, is we are forced to carefully evaluate the motivation for the definition of each class to group objects according to the most diagnostic spectral parameters (particularly considering the much wider spread of the SDSS dataset), consistency with previous classifications, and potential compositional interpretation. Additionally, fixing the boundary allows us to more easily use the classifications as a tool. We can use these classifications to determine the fraction of objects in each class and the mass of each taxonomic type across the Solar System.

5.2. Defining the class boundaries

We transform the apparent magnitudes from SDSS to reflectance values to directly compare with taxonomic systems based on reflectance data. We then subtract solar colors in each filter and calculate reflectance values using the following equation:

$$R_f = 10^{-0.4[(M_f - M_g) - (M_{f,\odot} - M_{g,\odot})]} \quad (2)$$

where (M_f) and $(M_{f,\odot})$ are the magnitudes of the object and Sun in a certain filter f , respectively, at the central wavelength of the filter. The equation is normalized to unity at the central wavelength of filter g using (M_g) and $(M_{g,\odot})$: the g magnitudes of the object and Sun, respectively. Solar colors used in this work are $r'-g' = -0.45 \pm 0.02$, $i'-g' = -0.55 \pm 0.03$, and $z'-g' = -0.61 \pm 0.04$ from Holmberg et al. (2006). Note that because we use solar colors in the Sloan filters we do not convert from the g' , r' , i' , z' filters (central wavelengths: $g' = 0.4686$, $r' = 0.6166$, $i' = 0.7480$, $z' = 0.8932 \mu\text{m}$) to standard g , r , i , z filters. As mentioned in Section 2.1, we do not use the u' filter because of the very large errors for this datapoint.

The classification of the dataset is based on two dimensions: spectral slope over the g' , r' , and i' reflectance values (hereafter gri-slope), representing the slope of the continuum, and $z'-i'$ color, representing band depth of a potential $1 \mu\text{m}$ band. We restrict the evaluation of the spectral slope to g' , r' , and i' filters only, excluding the z' filter because it may be affected by the potential $1 \mu\text{m}$ band. These two parameters (slope and band depth) are the most characteristic spectral distinguishers in all major taxonomies beginning with Chapman et al. (1975) because they account for the largest amount of meaningful and readily interpretable variance in the system.

We choose not to use a^* defined by Ivezić et al. (2002) or the first Principal Component (PC1) defined by Nesvorný et al. (2005) used in other works. a^* is the first principal component of the $r'-i'$ versus $g'-r'$ colors and PC1 is the first principal component of the measured fluxes of all five filters. To most effectively use Principal Component Analysis, the dimension with the greatest variance, slope in this case, should be removed before running PCA

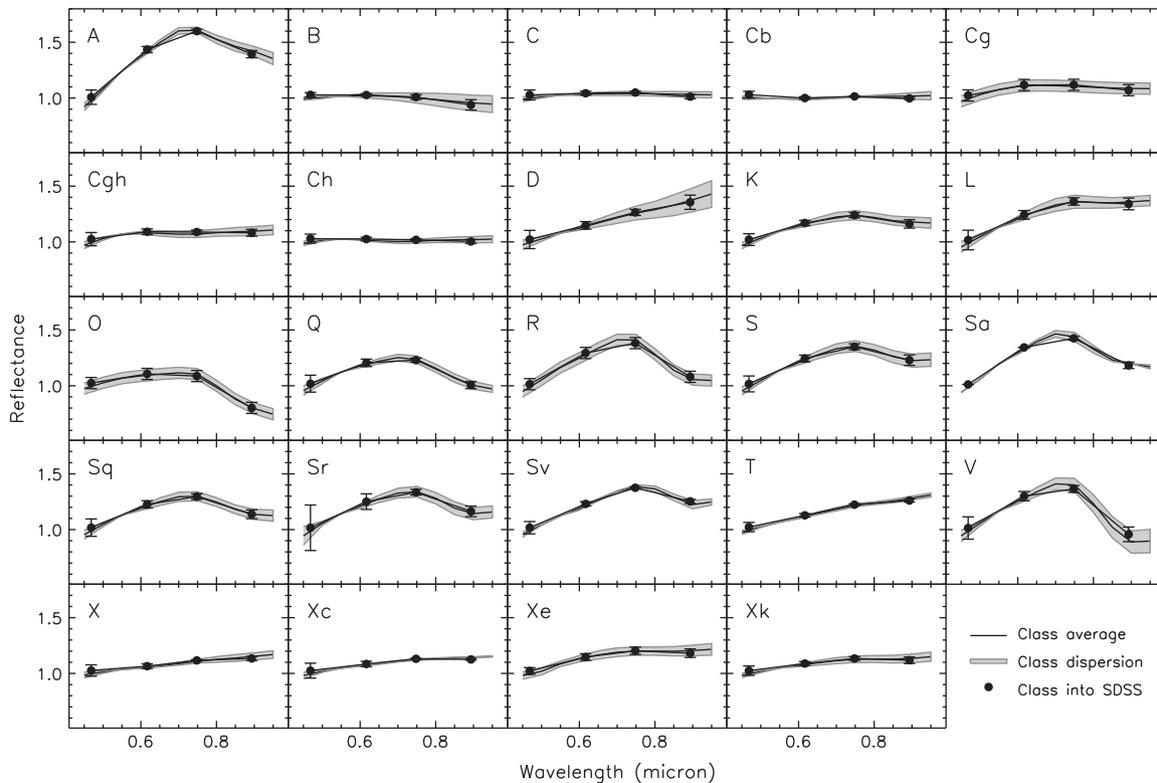


Fig. 4. Average Bus–DeMeo (DeMeo et al., 2009) spectra converted to SDSS colors used to define the classification boundaries. The black dots (with 1 standard deviation from the mean plotted) represent the average Bus–DeMeo spectra converted into SDSS colors. The u' filter is extrapolated from the data because the spectra do not cover those wavelengths (the u' filter is not used in the classification of SDSS data, however). The gray background plots the average spectrum plus one sigma for comparison with the colors. Because the Cg, O, and R classes are defined by a single object the standard deviation is set to 0.1. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

to increase sensitivity to more subtle variation (see discussion in Bus (1999)). We also disfavor the inclusion of the u' filter (used for PC1) as it adds significant noise to the data (Fig. 2). We find our slope parameter is reasonably well-correlated with a^* but not well-correlated with PC1, as expected from the use of u' photometry in PC1.

We base the classification on the 371 spectra used to create the Bus–DeMeo taxonomy (DeMeo et al., 2009), whose classes are very similar those of the Bus taxonomy (Bus and Binzel, 2002a), with a few classes removed. The variation among the reflectance spectra of the 371 asteroids used to define the Bus–DeMeo classes helped guide the boundary conditions of the present SDSS taxonomy. We convert all the spectra into SDSS reflectance values by convolving them with the SDSS filter transmission curves,² thus providing the average SDSS colors and standard deviation per class (see Fig. 4).

Because the SDSS data have a spectral resolution significantly lower than the Bus–DeMeo data set (see Fig. 4) and subtle spectral details are lost, we combine certain classes into their broader complex. The C-complex encompasses the region including C-, Cb-, Cg-, Cgh-, and Ch-types. The S-complex encompasses the S-, Sa-, Sq-, Sr-, and Sv-types. The X-complex includes X-, Xc-, Xe-, Xk-, and T-types. The classes that are maintained individually are A, B, D, L, K, Q, and V. While we distinguish all these classes based on the SDSS colors here, we slightly modify our use of some of these classes for this work (see Section 6.1). We do not classify the rare R- or O-type in this work, because there is significant overlap between O-types or R-types and other classes in the visible wavelength range, and they are particularly rare classes. The R class

would overlap the V class essentially spanning the shallower $z'-i'$ “band depth” region. We tested separating the R class, but the majority of the objects classified as R were located in the Vesta family.

While the Bus–DeMeo class averages are very useful as a guide, the system was based on a sample size three orders of magnitude smaller than present SDSS sample. The SDSS dataset therefore shows a much more continuous range of reflectance characteristics. To compare the two datasets, we plot the distribution of SDSS objects in $z'-i'$ color and gri-slope, with the 371 objects from the Bus–DeMeo taxonomy (Fig. 5). Furthermore, the figure shows the boundaries for each class defined in this work. We drew boundaries that best separated each class based on the position of the class centers and standard deviations based on the 371 spectra dataset. We visually inspected each boundary by plotting the spectral data on each side of the boundary and comparing them with the designated class to tweak the position of the line and best separate each class.

We strove to preserve the uniqueness of the more exotic classes, restricting A- and D-types to the outliers with the largest slopes, and Q- and V-types with the deepest bands. The B-type was defined to have both a large, negative gri-slope and a negative $z'-i'$ value. A list of all the boundaries is provided in Table 3. Classification is performed in decision tree form, where the gri-slope and $z'-i'$ value of the asteroid is compared with each region in the following order: C, B, S, L, X, D, K, Q, V, A. If the object falls in more than one class, it is designated to the last class in which it resides. As can be seen in Fig. 5, there are a handful of objects that reside outside the defined classes. We give these objects the designation “U”, historically used to mark unusual objects in a sample that do not fall near any class. We do not include these objects in

² <http://www.sdss.org/dr7/instruments/imager/>.

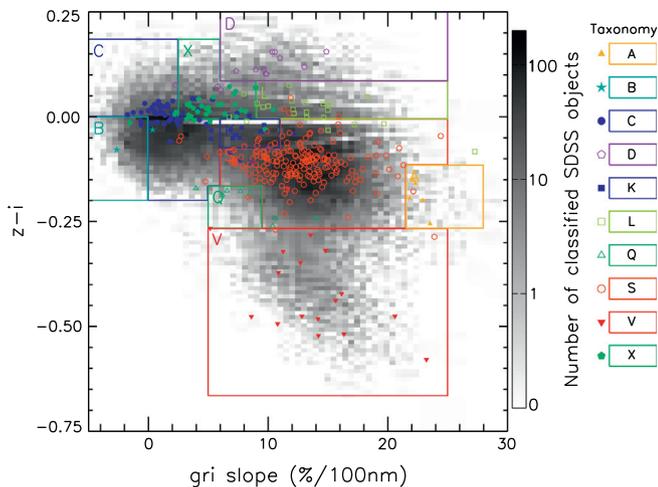


Fig. 5. Boundaries used to classify SDSS data into taxonomic classes. The colored points are the spectra from the Bus–DeMeo taxonomy (DeMeo et al., 2009) converted to SDSS colors. In the background, the density of the number of objects from MOC4 are plotted to show the dispersion of the SDSS data. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 3

Table of classification boundaries. The classification is performed following the order: C, B, S, L, X, D, K, Q, V, A.

Class	Slope (%/100 nm)		$z'-i'$	
	(min)	(max)	(min)	(max)
A	21.5	28.0	-0.265	-0.115
B	-5.0	0.0	-0.200	0.000
C	-5.0	6.0	-0.200	0.185
D	6.0	25.0	0.085	0.335
K	6.0	11.0	-0.075	-0.005
L	9.0	25.0	-0.005	0.085
Q	5.0	9.5	-0.265	-0.165
S	6.0	25.0	-0.265	-0.005
V	5.0	25.0	-0.665	-0.265
X	2.5	9.0	-0.005	0.185

our study. Most of these extreme behaviors are likely due to problems with the data even though no flags were assigned (see details in Section 2.1). Follow-up observations could determine whether the objects really are unique.

5.3. Determining a single classification for multiple observations

Of the many observations in the SDSS MOC that remained after we applied cuts on the photometric precision (58,607, see Section 2.1), many were actually the same object observed more than once. The number of unique objects in our sample is 34,503. For some of these objects, not all observations fell into the same class. Because we seek to categorize each object with a unique class, we use the following criteria to choose a single class for any object that has multiple observations that fall under multiple classifications (5401 asteroids, i.e., 15.7% of the sample):

- The class with the majority number of classifications is assigned (2619 asteroids, i.e., 7.6% of the sample)
- If two classes have equal frequency and one of them is C, S, or X we assign the object to C, S, or X, continuing the philosophy of remaining conservative when assigning a more rare class (1867 asteroids, i.e., 5.4% of the sample)
- If the two majority classes are C/S, X/C, S/X (or three competing classes of C/S/X) we assign it to the U class and disregard those objects in the distribution work (919

asteroids, i.e., 2.6% of the sample). We prefer to keep the sample smaller, rather than contaminate it with objects that we have randomly chosen a classification among C, S, or X and thus possibly bias the sample.

- For objects that are assigned multiple classes but none is either the majority or C, S, or X is assigned to the U class.

Among the largest asteroids, particularly those between H magnitudes of 9 and 12, several asteroids observed by the SDSS had taxonomic classes from previous spectroscopic measurements. The classification based on SDSS and previous work were generally consistent, but in cases that differed, we assigned the asteroid to the class determined by spectroscopic measurements.

5.4. Caution on taxonomic interpretation

One must be careful when interpreting the classifications presented here. First, the resolution of the SDSS data are significantly lower than the spectra to which they are compared. Second, the fact that we find multiple classifications for multiply observed objects suggests there is a larger uncertainty in the data than expected. Third, for many classes (particularly L, S, Q, A), the visible data can only suggest the presence of a 1 μ m band, but do not actually predict the depth or shape of that band (for more detail see DeMeo et al., 2009; DeMeo, 2010). This is important because, for example, a spectrum might look closer to a K- or an L-type in the visible range, but near-infrared data could place them more confidently in the S-class (or vice versa).

Each class is meant to be representative of a certain spectral characteristic, but with limited wavelength coverage and limited resolution, there is some degeneracy. For example, the Q-class defined here represents objects with a low slope and moderate 1 μ m band depth. We do not suppose that all objects classified as a Q-type are young, fresh surfaces as is typically associated with the Q class. Careful follow-up observations are important to make such a claim.

Defining boundaries for C, X, and D-types is not easy because they are distinguished only by slope and there is a continuous gradient of slope characteristics. This problem is not unique to the SDSS dataset. The boundary between each type is somewhat arbitrary. The difference between a C-type of slope zero and a D-type with a high slope is meaningful, however we do not yet know how to interpret the significance of these spectral differences. It is likely that there is some contamination between C- and X-types with our classification scheme, though it is unlikely that much contamination exists for example between C- and D-types that are more easily distinguished.

5.5. Verification of our classifications

With a unique class assigned to each object in our dataset we can now evaluate the robustness of our classifications. First, we compare the classification of each asteroid with the results of Carvano et al. (2011) available on the Planetary Data System (Hasselmann et al., 2011). Because their classification is based on the same dataset, it is not an entirely independent check. However, their classification method is different so consistency between the two supports both methods. Fig. 6 graphically compares the two classifications. We list the classification differences that are generally compatible but represent the different choices each method made. We find the two classifications quite consistent. Of the major classification differences between the two methods we suspect some are due to boundary condition differences and others are due to Carvano's inclusion of the u' filter, which we exclude in our work (see Section 2.1).

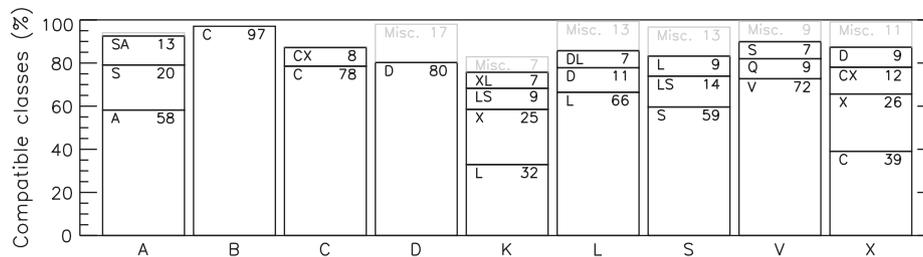


Fig. 6. Comparison of classifications in this work to those of Carvano et al. (2011). For each class in our work a bar represents how those objects are classified in the Carvano system. Some objects were given two letters by Carvano given in the PDS archive (Hasselmann et al., 2011). We categorize according to the most numerous classes assigned by Carvano compared to this work. All “compatible” classes are shown since they are relatively in agreement based on classes that border others. These highlight the small but compatible differences between the classifications. Miscellaneous includes other classes we feel are compatible but make up a small percentage of the sample. All B-types in our work are classified as C-type by Carvano because they do not make a distinction between the two. The small unlabeled fraction represents mismatches where our work and Carvano’s get significantly different results. The right side of each bar labels the percent of the total each Carvano class represents.

Second, we retrieved the albedo of the asteroids as determined from IRAS, AKARI, and WISE data (Tedesco et al., 2002; Ryan et al., 2010; Usui et al., 2011; Masiero et al., 2011, 2012; Grav et al., 2011, 2012a,b). We found 17,575 asteroids (out of 34,503, i.e., 51%) with albedo determinations. We present in Fig. 7 the distribution of albedo for each class, and the average values in Table 4. The agreement between the average albedo per Bus–DeMeo class from previous work (see Table 1) and of the asteroids classified from SDSS colors gives confidence in our capability to assign a relevant class to these asteroids. One of the greatest differences is the albedo of the B class.

We have separated the spectra of objects with negative slopes into the B class using SDSS colors as has traditionally been done in taxonomic systems. This separation can be a useful indicator of spectral differences between B and C classes. The average albedo for B-types classified from spectroscopic samples is significantly higher than for C-types (see Table 1) suggesting a compositional difference. However, the average albedo of B-types in our SDSS sample is similar to that for C-types so we caution that the B-types classified from spectroscopic surveys may not be fully representative of the B-types in our sample.

6. Building the compositional distribution

6.1. Additional taxonomic modifications

Keeping in mind the cautions mentioned in Section 5.4, for the taxonomic distribution work presented here we apply slight modifications to the classes. First, we note a significant over abundance of S-types in the Eos family. This is due to the similarity of S- and K-type spectra using only a few color points and the visible-only wavelength range. We thus reclassify all S-type objects to K-type within the Eos family (defined by the family’s current orbital elements $a \in [2.95, 3.1]$, $i \in [8^\circ, 12^\circ]$, and $e \in [0.01, 0.13]$). Reviewing this change shows that the background of S-types is now evenly distributed, no longer showing a concentration within the Eos family. Additionally, for this study we group Q-types with the S-types because they are compositionally similar (Binzel et al., 2010; Nakamura et al., 2011).

In the previous section we discussed the albedo differences between B-types in our sample and B-types from other work. While future work may want to focus specifically on objects with negative slopes, in this we choose to merge B-types with C-types classified by the SDSS dataset.

Our SDSS observations classify some Hildas and Trojans as K- and L-types. Careful examination reveals that for the K- and L-type objects that are near the border the X and D classes, the spectra could also be consistent with X and D. These Hilda and Trojan K- and L-types that have multiple observations are also classified X

and D. For example, the Centaur (8405) Asbolus has a very red visible spectral slope (Barucci et al., 1999), categorizing it as a D-type. Eight SDSS observations place this object in the L class and four in the D class. This difficulty is partially due to the degeneracy of the visible wavelength data. The Bus Ld class that is intermediate between the L and D classes does not remain an intact definable class when near-infrared data are available (DeMeo et al., 2009). There are four SDSS L-type Hildas and Trojans with albedos all of which are below 0.08 further suggesting that these objects are not characterized by what the K and L classes are compositionally meant to represent. We therefore choose the more conservative option to reclassify the Hilda and Trojan K- and L-types. The K-types that have slopes more consistent with the X class are relabeled as X, while the L-types have slopes more consistent with D-type and are relabeled as D.

Among the Hungarias, a population of small ($H > 13$) C-types is seen. Upon closer inspection, all (8) of the small C-types with WISE data have extremely high albedos (0.4–0.9), suggesting they are actually E-types (the high albedo group of X-types). This is unsurprising, as the Hungaria region is known to contain a large population of high albedo E-type asteroids. We thus correct our Hungaria sample by assuming all small C-types are incorrectly classified, and remove those with H magnitudes greater than the E-type cutoff from our sample. We expect some overlap between C-types and X-types (E, M, P) in other regions of the belt as well (as addressed in Section 5.4) although the classification of X v. C should be more balanced.

While we make these modifications for the objects in the SDSS sample we do not make changes to the large objects classified spectroscopically from previous work.

6.2. Discovery completeness

While we select the subset of SDSS data where the survey is efficient, the dataset not complete. The information from the SDSS dataset must be applied to all existing asteroids in the same size range. The Minor Planet Center (MPC) catalogues all asteroid discoveries. Here we assess discovery completeness.

In Fig. 8, we plot the cumulative number of discoveries in the outer belt for every two years of the past 10 years (to 2013–01–01). We derive a limiting magnitude for the completeness of the MPC database of $H = 16$, 15 and 14.5 (diameter of about 2, 3, and 4 km) for the inner, middle, and outer belt, respectively. We determine the completeness of small asteroids in each section of the main belt by extrapolating the size of the population using a power law fit to each region of the main belt (shown in Fig. 1). The difference between the currently observed populations and the extrapolated populations derived from these power laws provide the expected number of asteroids to be discovered at each

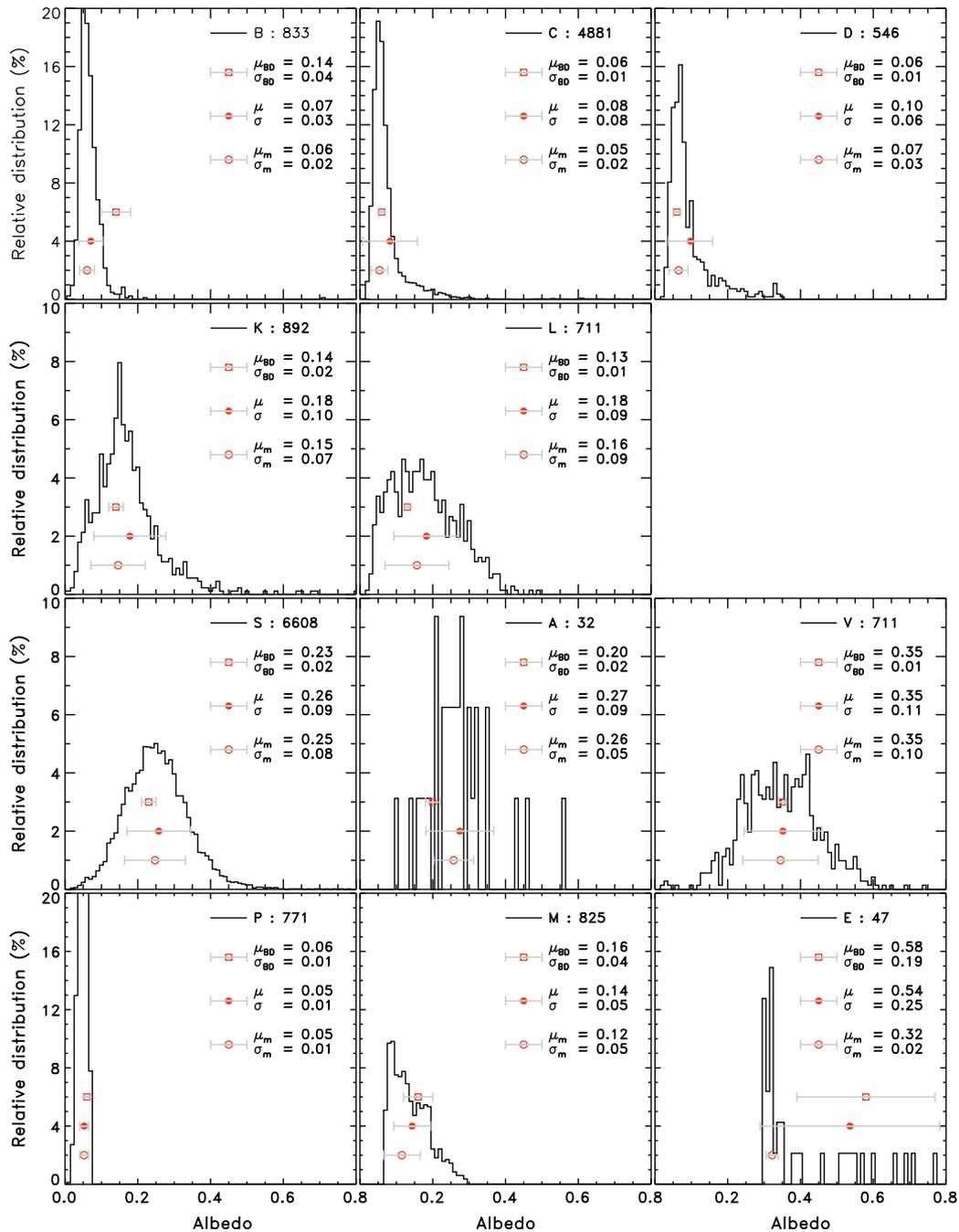


Fig. 7. Relative distribution of albedo for each class. For each class, we report the number of asteroids with albedo estimates, and the average albedo with its standard deviation (μ , σ , open circle), together with the mode of the histogram (μ_m , σ_m , filled circles). We also report the average albedo of the asteroids in the Bus–DeMeo sample (μ_{BD} , σ_{BD} , square symbol, see Table 1).

size range. The power law indices we find for the IMB, MMB, and OMB (determined over the H magnitude range 14–16, 13–15, and 12–14.5) are -2.15 , -2.57 , and -2.42 , respectively. These power law indices agree with other fits to the observations (Gladman et al., 2009) as well as with the theoretical index calculated assuming a collision-dominated environment (Dohnanyi, 1969).

For almost all H magnitudes in our sample we are nearly discovery complete. For the smallest size we use a power law function to determine completeness. In the H = 15–16 magnitude range, we are 100%, 85%, and 60% complete in the IMB, MMB, and OMB respectively. When applying the taxonomic fractions to the MPC sample of known asteroids we add a correction factor to account

the 15% and 40% of objects that have not been discovered in the middle and outer belt in the H = 15–16 range. For Cybeles, Hildas, and Trojans we do not extrapolate to determine sample completeness because there is far too much uncertainty in the size distribution of those populations due to fewer discoveries. We have not corrected these populations. The completeness of our dataset can be evaluated on Fig. 1.

There are undoubtedly still many objects yet to be discovered, especially at sizes smaller than we cover in this work. For reference, we explore the total mass these undiscovered objects are expected to represent. The largest objects represent the overwhelming majority of the mass in the main belt. In fact, the asteroids from the spectral surveys (particularly H < 10) represent

Table 4

Average albedo of each class based on the 17,575 objects in our SDSS dataset that had calculated albedos (51% of our dataset). The results are consistent with previous albedo averages (Tables 1 and 2) strengthening the robustness of this work.

Class	N_{objects}	Average	Mode
A	32	0.274 ± 0.093	0.258 ± 0.055
B	833	0.071 ± 0.033	0.061 ± 0.021
C	4881	0.083 ± 0.076	0.054 ± 0.023
D	546	0.098 ± 0.061	0.065 ± 0.026
K	892	0.178 ± 0.099	0.146 ± 0.075
L	711	0.183 ± 0.089	0.157 ± 0.088
S	6565	0.258 ± 0.087	0.247 ± 0.084
V	711	0.352 ± 0.107	0.345 ± 0.104
E	47	0.536 ± 0.247	0.322 ± 0.016
M	825	0.143 ± 0.051	0.115 ± 0.051
P	771	0.053 ± 0.012	0.053 ± 0.012

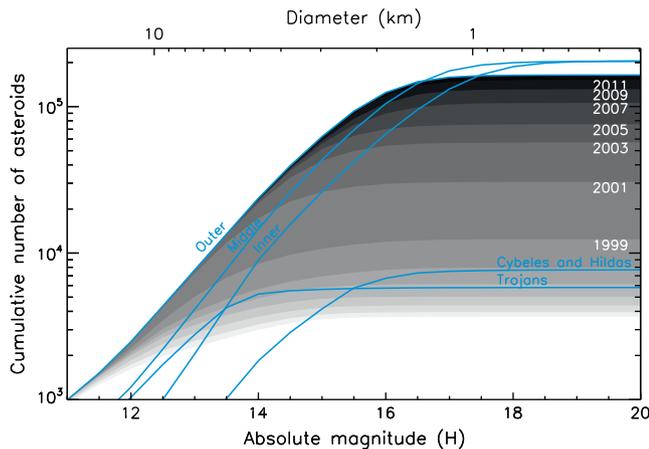


Fig. 8. Discovery completeness through 2013. For the outer belt, we plot the cumulative distribution as function of time up to 2013 January 01 (shades of gray), showing the evolution of the completeness limit to smaller (higher H) objects. We derive a limiting magnitude for the completeness of the MPC database of $H = 16$, 15 and 14.5 (diameter of about 2, 3, and 4 km) for the inner, middle, and outer belt, respectively.

97% of the mass (assuming a mass of 30×10^{20} kg for the entire main belt, Kuchynka et al., 2013).

We calculate the undiscovered mass (assuming a general density of 2.0 g/cm^3 and an albedo of 0.18, 0.14, and 0.09 for the inner, mid, and outer main belt, based on WISE measurements, see Mainzer et al., 2011) up to H magnitude of 22 to be 5.7×10^{12} , 4.8×10^{13} , and 1.6×10^{14} kg for the IMB, MMB, and OMB, that each contain a total mass (with the same generic albedo and density assumptions) of 6.2×10^{20} , 1.3×10^{21} , and 7.1×10^{20} kg, with a total of 26×10^{20} kg. Therefore, although hundreds of thousands of asteroids will still be discovered and they will provide valuable information about asteroids at small size scales, their expected contribution in terms of mass is minuscule (below the part per million level).

6.3. Applying the SDSS distribution to all asteroids

After applying data quality cutoffs, H-magnitude cutoffs, and taxonomic classifications, we can now calculate the number of SDSS objects in each class according to size and distance. We use H magnitude bins of 1 magnitude ranging from 3 to 16 (though each class has its appropriate H magnitude cutoff listed in Table 2). The semi-major axis bins applied are 0.02 AU wide ranging from 1.78 to 5.40 AU. Only asteroids among Hungarias, the main belt, Cybeles, Hildas and Trojans are included in this study covering the distances 1.78–2.05, 2.05–3.27, 3.27–3.7, 3.7–4.2, and 5.05–5.40 AU, respectively. Near-Earth objects, comets, Centaurs,

transneptunian objects and any other objects outside the mentioned zones were excluded. We calculate the number of objects in each bin and the fraction of each class in each bin (F_i , where i is the taxonomic class). For example, for objects with H magnitudes between 13 and 14 and semi-major axes between 2.30 and 2.32 AU, we might find 60% of the objects are S-type ($F_s = 0.6$), 20% are C-type ($F_c = 0.2$), 20% are X-type ($F_x = 0.2$). Figs. 9 and 10 show the bias-corrected and biased fraction of objects. The biased view of the asteroid belt shows a dominance of S-types (by number) out to nearly 3 AU because of the inclusion of the abundant smaller, higher albedo bodies (whose small, dark counterparts, the C-types, were not observed). The bias-corrected version demonstrates that instead, the S-types and C-types alternate dominating by number throughout the belt. Asteroid families play an important role in these figures since they contribute large numbers of taxonomically similar objects.

Albedo data enable the separation of X-types into three subgroups: E, M, P (Tholen, 1984). Since albedo data are not available for every single spectral X-type, we calculate the fraction of E, M, and P for each region: Hungaria, Inner, Middle, Outer, Cybele, Hilda, Trojan. This fraction is calculated based on ~ 2000 X-type objects in our sample with albedo measurements (from a total of 2500 X-types) from IRAS, AKARI, and WISE (Ryan et al., 2010; Usui et al., 2011; Masiero et al., 2011). See Fig. 11 for the bias-corrected distribution of the E, M, and P types across the main belt that was used to extrapolate the X-type EMP fraction for our entire dataset. Among Hungarias the sample is entirely E-type as expected. There are an insignificant number of E-types among the other regions (though we note a bias against observing high visible albedo objects in mid-infrared wavelength ranges). The fraction of all bias-corrected X-types that are M in each region are: 0.00, 0.58, 0.44, 0.35, 0.28, 0.08, and 0.17, respectively. The fraction for P-types is thus one minus the M-type fraction, except for the Hungaria region where it is also zero. Among Trojans we find that 0.17 (1 out of 6) X-types have an M-type albedo, however because of large uncertainty due to a small sample we assume the same fraction for Trojans as Hildas (0.08).

We now know the relative abundance of each taxonomic type at each size range and distance determined from the SDSS dataset with and adjustment for the division of E, M, and P types from the X class. However, at many size ranges the SDSS only observed $\sim 30\%$ of the total asteroids that exist at that size and distance. As long as we only use a size range in which asteroid discovery is essentially complete or make a correction for discovery incompleteness, we can apply these fractions to the entire set of known asteroids at these sizes from the Minor Planet Center to determine the distribution of taxonomic type across the main belt according to number, surface area, volume, and mass.

When calculating the number of objects, surface area, volume, or mass at each size range and distance we use two different methods. For the largest asteroids with $H < 10$ where our SDSS sample is complete, we calculate the surface area, volume, or mass for each asteroid using that body's H magnitude, albedo (or average albedo for its taxonomic class when not available), and average density (Carry, 2012) for that taxonomic class.

For objects with $10 < H < 13$, where our sampling is not complete, we use the following method. The surface area, volume, or mass is calculated for an object using the H magnitude, average albedo and average density for that class. That value is multiplied by the number of objects of that class in that (size and distance) bin, N_{bin} , and by the fraction (F_i) of objects of that class from SDSS: $N_i = N_{bin} \times F_i$.

For objects with $H > 13$ we have the added complication that we cannot directly apply our fraction to the total number of known objects because our fraction of each type at each size from SDSS

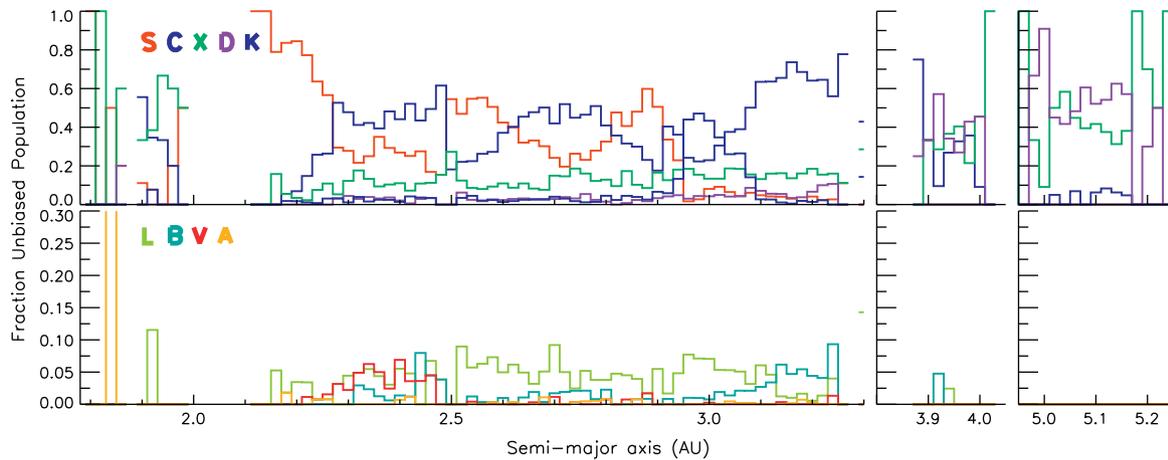


Fig. 9. The bias-corrected fraction of each class in each 0.02 AU bin according to SDSS data (each bin summing over all classes equals 100%). All objects are 5 km or larger. The distribution in this figure is dominated by smaller objects (H of 13–15.5). Because we are plotting by number of asteroids, the collisional families play an important role in this figure (e.g., the Vestoids in the inner belt).

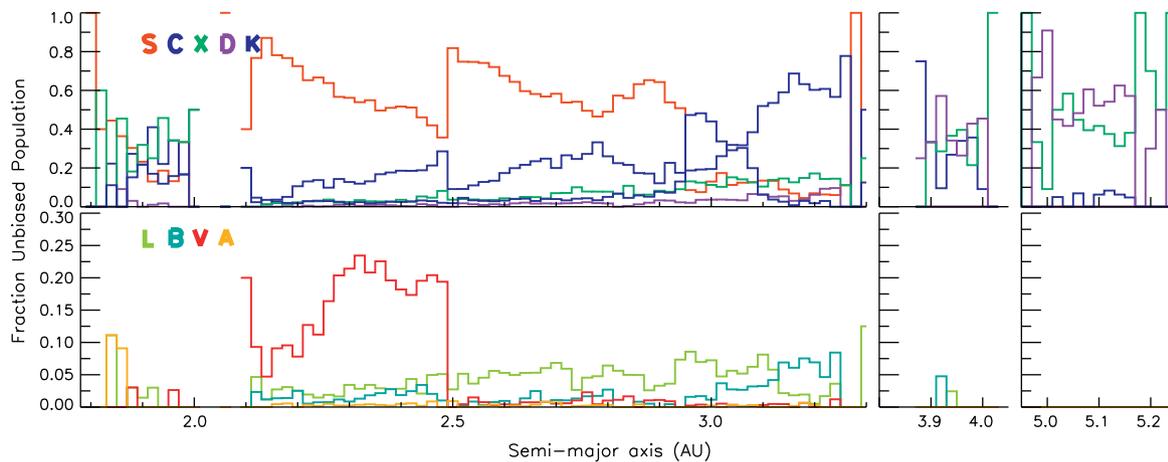


Fig. 10. The observed fraction of each class in each bin according to SDSS data (each bin summing over all classes equals 100%). In this case we did not cut the sample at a particular size range. The smaller, brighter S-types are more prevalent everywhere, and small, bright V-types make up nearly 20% of the second half of the inner belt. In this case we are plotting S-types smaller than 5 km whereas we are not sampling the darker C-types of similar size. The difference between this figure and Fig. 9 demonstrates the importance of correcting a sample for observational biases.

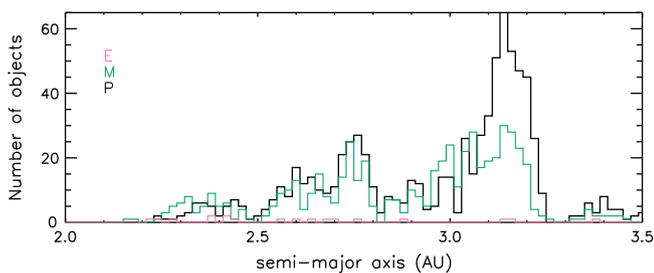


Fig. 11. The bias-corrected distribution of E, M, and P-type asteroids across the main belt based on the 1500 X-types in our sample (including spectral surveys and SDSS) with WISE, AKARI, or IRAS albedo determinations (Ryan et al., 2010; Usui et al., 2011; Masiero et al., 2011). These objects are used to determine the relative fraction of M to P types among X class objects in each zone of the belt. Because only a subset of our SDSS X-types had albedos available we applied the fraction of M/P in each region to our entire X-type dataset. In the area near 3.0 AU we remove all SDSS objects classified as X-type in the Eos family. Because of the spectral similarity between X and K-types in SDSS colors, many K-types Eos family objects were classified as X (see Section 6.1 for discussion on classification ambiguity).

is calculated with certain (higher albedo) classes removed. We thus must also calculate the fraction of objects in the SDSS database that were kept, F_{kept} , (i.e., those that were not removed because they are

smaller than ~ 5 km) for H magnitude bins $H = 13, 14,$ and 15 . For all other size bins F_{kept} is equal to 1. The number of objects of a certain class (N_i) can be determined by the total number of discovered objects in that bin (N_{bin}) multiplied by the fraction of objects in that class (F_i) and by the fraction of objects in that bin that are kept (F_{kept}), thus $N_i = N_{bin} \times F_i \times F_{kept}$.

Previously in this section, we calculated the bias-corrected fraction of E, M, and P types in each zone, although, as above, we cannot apply this true (bias-corrected) correction factor to the observed (biased) MPC dataset. For the H bins 14 and 15 where some M-types were removed due to size we calculate the fraction of (M + P)-types kept in each region. The fraction for $H = 14$ is 0.60, 0.67, 0.64, 0.79, for the IMB, MMB, OMB, and Cybeles and 0.22, 0.39, 0.41, 0.70 for $H = 15$. There are no small objects in our sample to be removed in the Hilda and Trojan regions so the fraction kept is unity.

Finally, if we simply assume an average H magnitude for each bin (say 12.5 for the $H = 12$ bin) we could potentially over- or underestimate the surface area, volume, or mass, depending on the H magnitude distribution of objects in that bin. We thus calculate the number of objects in each 0.1 H magnitude sub-bin and apply the same class fractions to each for accuracy.

7. The compositional makeup of the main belt

7.1. Motivation for number, surface area, volume, and mass

Previous work calculated compositional distribution based on the number of objects at each distance (e.g. Chapman et al., 1975; Gradie and Tedesco, 1982; Gradie et al., 1989; Mothé-Diniz et al., 2003). This was not unreasonable because those datasets included only the largest objects, often greater than 50 km in diameter.

If we restrict our study to the number of asteroids, our views would be strongly influenced by the small asteroids. There are indeed more asteroids of small size than large ones. This is the result of eons of collisions, grinding the asteroids down from larger to smaller. The size–frequency distribution of asteroids (Fig. 1) can be approximated by a power-law, and for any diameter below 20 km, there are about 10 times more asteroids with half the diameter. The amount of material (i.e., the volume) of the two size ranges is however similar: if there are n asteroids of a given diameter D , there are about $10n$ asteroids with a diameter of $D/2$, each with a volume 8 times smaller, evening out the apparently dominating importance of the smaller sizes. Ceres alone contains about a third of the mass in the entire main belt using a mass of 30×10^{20} kg for the main belt (Kuchynka et al., 2013), and 9×10^{20} kg for Ceres (from a selection of 28 estimates, see Carry, 2012), and yet it is negligible (1 out of 600,000) when accounted for in a distribution by number. Therefore, the relative importance of Ceres in the main belt can change by orders of magnitude depending on how we look at the distribution.

The study of the compositional distribution by number is perfectly valid and is useful for size–frequency distribution studies and collisional evolution. For studies of the distribution of the amount of material, it puts too much emphasis on the small objects compared to the largest. A simple way to balance the situation is to consider each object weighted by its diameter. This opens new views on asteroids: we can study how much surface area of a given composition is accessible for sampling or mining purposes (Sec-

tion 7.2), or how much material accreted in the early Solar System has survived in the Belt (Sections 7.3 and 7.4 for the distributions by volume and mass).

7.2. Asteroid distribution by surface area

To estimate the surface area of each asteroid, we need first to estimate its diameter D . For that we use the following equation from Pravec et al. (2007, and references therein):

$$D = \frac{1329}{\sqrt{a}} 10^{-0.2H} \quad (3)$$

where H is the absolute magnitude (determined by the SDSS survey) and a is the albedo. For the largest asteroids ($H < 10$) we use the object's calculated albedo from WISE, AKARI, or IRAS. For small asteroids, and large ones for which no albedo is available, we use the average albedo for that object's taxonomic class, see Section 2.2. The equation above provides a crude estimation of the diameter only. Evaluation for a particular target should be considered with caution, the absolute magnitude and albedo being possibly subject to large uncertainties and biases (e.g., Romanishin et al., 2005; Mueller et al., 2011; Pravec et al., 2012). We can nevertheless make good use of this formula for statistical purposes: the precision on the diameters is indeed rough but seemingly unbiased (Carry, 2012). With a diameter D determined for each asteroid, we estimated their individual surface area S by computing the area of a sphere of the same diameter: $S = \pi D^2$. The surface area distribution is presented in Fig. 12.

The total surface area per bin ranges from 10^3 km² in the Hungarias to 10^6 in the main belt. Viewing the distribution with respect to surface area we can immediately notice the relative importance of larger bodies. Ceres and Pallas are represented by the blue peak near 2.75 AU and Vesta by the red peak near 2.35 AU. Additionally the E-types in pink, distributed throughout the main belt, are each only one or two asteroids.

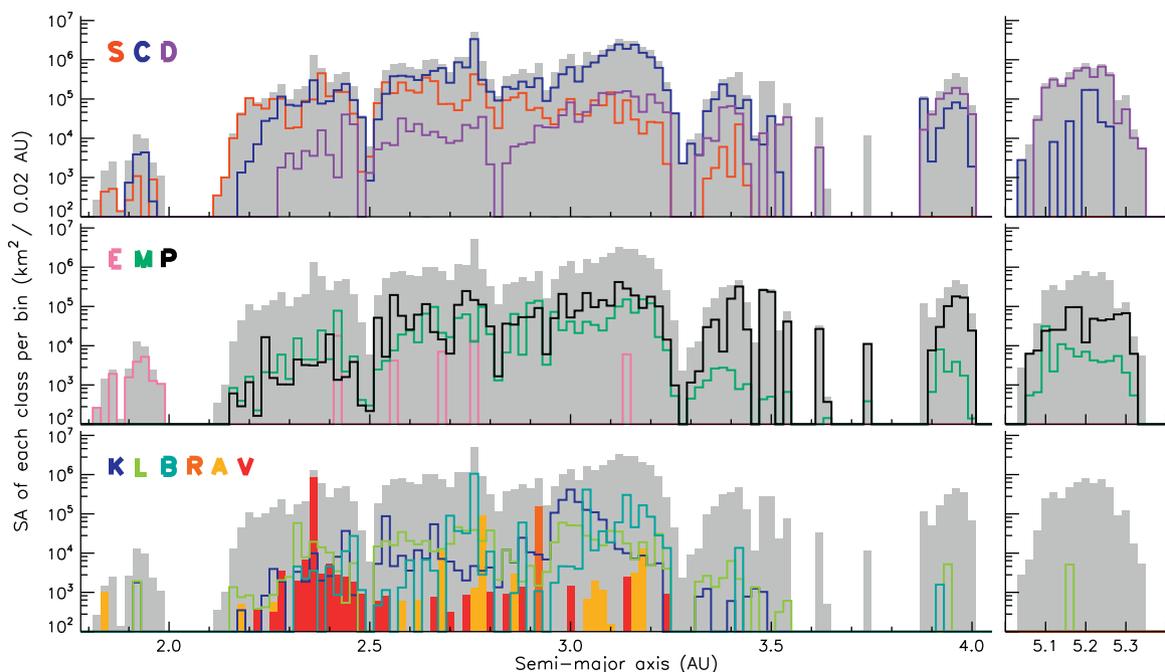


Fig. 12. The surface area (km²) of each taxonomic class in each 0.02 AU. The y-axis scale is logarithmic to include all classes on the same scale. All objects are 5 km or larger. While we do not classify R-types in our SDSS dataset, the one R-type in this plot is (349) Dembowska, from the spectroscopic surveys.

7.3. Asteroid distribution by volume

While the real value we seek is mass, because the density contributes significant uncertainty to the mass calculation we also present the distribution according to volume of material which gives similar results but is not affected by density uncertainties.

To evaluate the amount of material in the main belt, for each taxonomic class, we estimate the volume \mathcal{V} of all the asteroids by computing the volume of a sphere of the same diameter: $\mathcal{V} = \pi D^3/6$. We use the same method to calculate the total volume distribution by applying SDSS taxonomic fractions to the MPC dataset as described in Section 6.3. By looking at the compositional distribution in terms of volume instead of numbers, most of the issues described in Section 7.1 are addressed. Indeed, if there are about 2500 asteroids with a diameter of 10 km in the main belt, their cumulated volume is 300 times smaller than that of Ceres, re-establishing the proportions. The conversion from numbers to volume also corrects our sample for an overrepresentation of the contribution by collisional families (when viewed by number). Indeed, a swarm of fragments is released during every cataclysmic disruptive event, “artificially” increasing the relative proportion of a given taxonomic class locally (e.g., the Vestoids in the inner belt, see Fig. 10). Here, we are accounting for all the material of the family as if put back together again.

We present the distribution of taxonomic class by volume in Fig. 13. The distribution is the same as for surface area, but with the y-axis stretched because volume is proportional to diameter cubed while surface area is proportional to diameter squared. Asteroid distributions by volume were first presented by Consolmagno et al. (2012).

7.4. Asteroid distribution by mass

Ultimately, the mass is the physical parameter we seek that provides insights on the distribution of material in the Solar System. To precisely measure the mass of each asteroid we would need a fleet of missions to fly by each asteroid. Barring that as an option in the foreseeable future, to *estimate* the mass of each asteroid we need an

approximate density together with the estimated volume determined above. The density is the least well-constrained value used in this work because these measurements are extremely difficult to obtain (see discussion in Britt et al., 2002, 2006; Carry, 2012). Nevertheless, the study of meteorites tells us that the available range for asteroid density is narrower than it may seem. Indeed, no meteorite denser than 7.7 g/cm^3 has ever been found, and most of the meteorites cluster in a tight range, from 2 to 5 g/cm^3 (see Consolmagno and Britt, 1998; Consolmagno et al., 2008; Britt and Consolmagno, 2003; Macke et al., 2010, 2011, and references therein), with the exception of iron meteorites above 7 g/cm^3 (see the summary table in Carry (2012)). This range may be wider, especially at the lower end, for asteroids due to the possible presence of voids in their interiors, such as the low density of 1.3 g/cm^3 found for Asteroid (253) Mathilde (Veveřka et al., 1997). However, even if we assign an incorrect density to an asteroid, the impact on its mass will remain contained within a factor of 4 at the very worst. The impact may even be smaller as the typical densities of the most common asteroid classes (i.e., C and S) are known with better accuracy (Carry, 2012).

The uncertainty on the density will therefore affect the distribution in a much lesser extent than equal weighting of bodies according to number. Of course, any uncertainty on any of the parameters will sum up in the total uncertainty. However, we are confident that the trends we discuss below are real: both the discovery completeness, diameter estimates, and average albedo and density per taxonomic class have become more and more numerous and reliable over the last decade.

To calculate the distribution by mass we apply the average density of each class (Table 2, Section 2.3) and multiply that by the volume determined in the previous Section. For Ceres, Vesta, Pallas, and Hygiea, the four most massive asteroids we include their measured masses (9.44 , 2.59 , 2.04 , and $0.86 \times 10^{20} \text{ kg}$, from Carry (2012), Russell et al. (2012)) each accounting for about 31%, 9%, 7% and 3% of the mass of the main belt, respectively (using a total mass of the belt of $30 \times 10^{20} \text{ kg}$, Kuchynka et al. (2013)).

The distribution of mass is presented in Fig. 14 The fractional distribution of each class throughout the belt is given in Fig. 15.

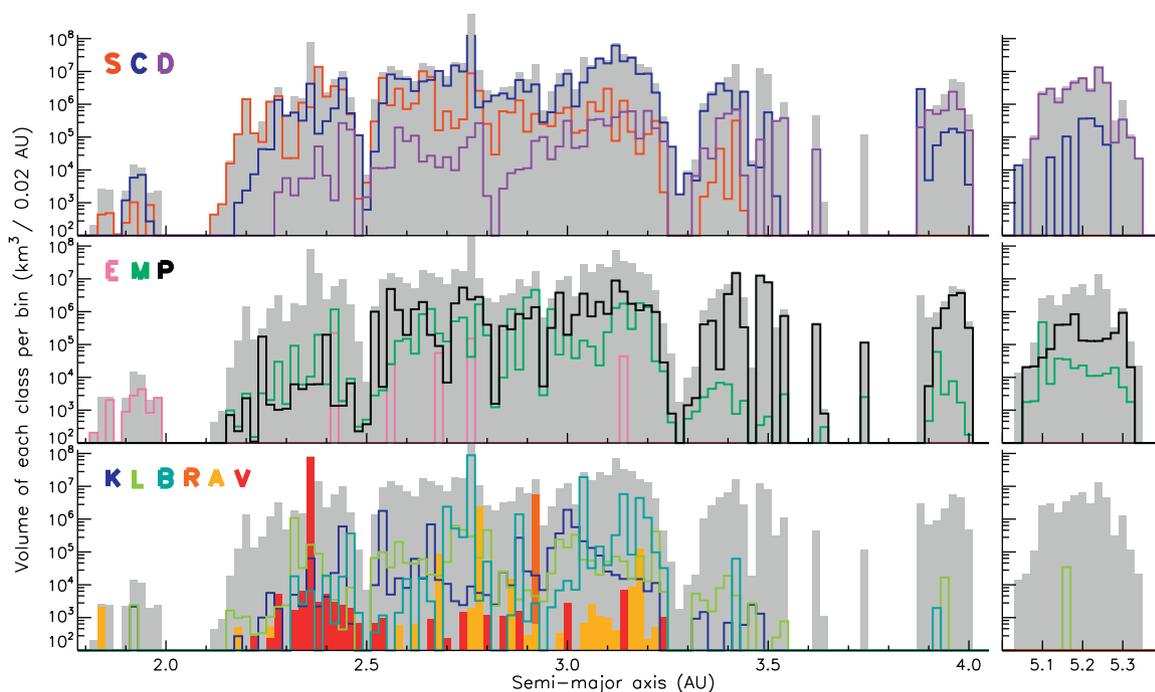


Fig. 13. The volume (km^3) of each taxonomic class in each 0.02 AU. The y-axis scale is logarithmic to include all classes on the same scale. All objects are 5 km or larger.

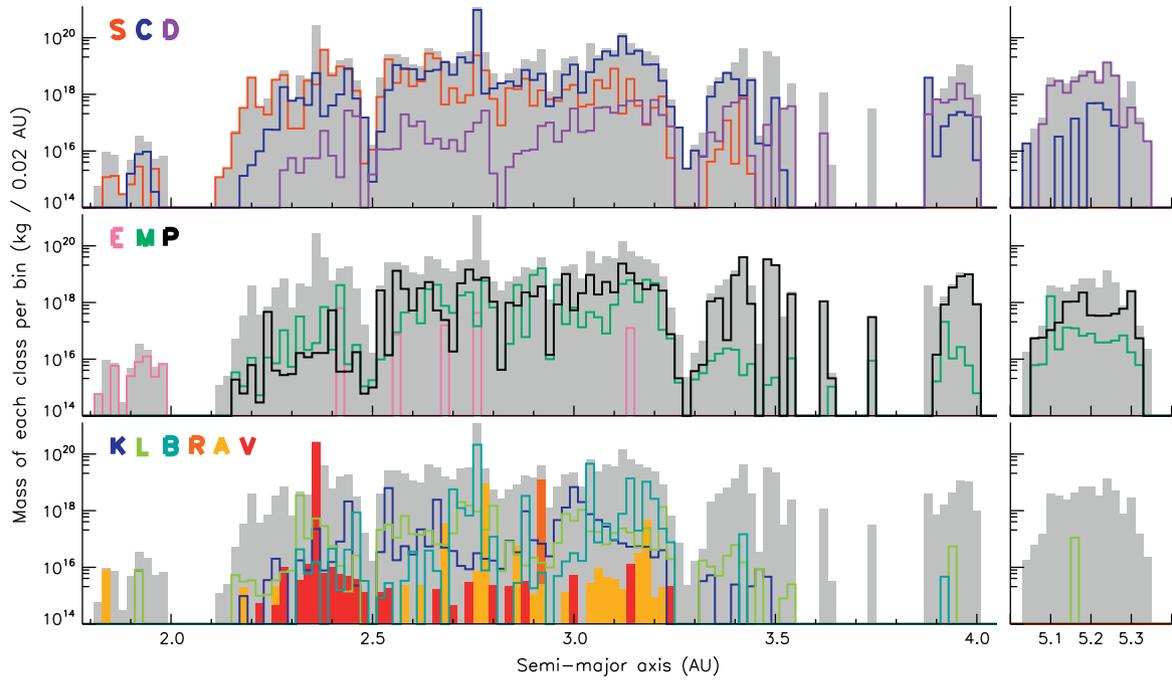


Fig. 14. The mass (kg) of each taxonomic class in each 0.02 AU bin. All objects are 5 km or larger.

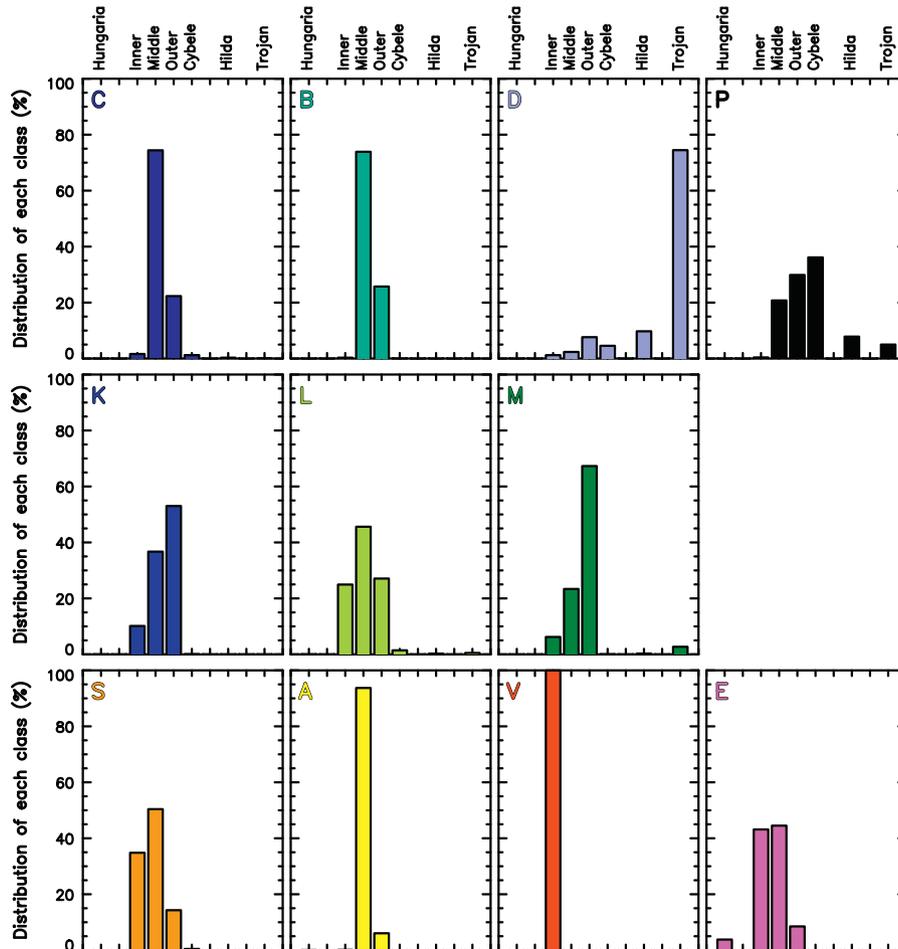


Fig. 15. The fractional mass distribution of each class across the belt. The total of each class across all zones sums to 100%.

Again, we find the general trends to be similar to volume and surface area. The difference in the case of mass is that the relative abundance of the taxonomic types have changed. Because S-types are generally denser than C-types by a factor of roughly 2 we see S-type material is more abundant relative to C than in our previous plots. Because in many cases the relative abundance of different taxonomic types already vary by an order of magnitude or more, we do not see drastic relative abundance changes. For example, C-types contribute more mass to the outer belt than S-types even though their relative abundance by mass is closer than by volume.

7.5. Search for S-types among Hildas and Trojans

We note a sharp cliff at the edge of the outer belt delineating the limit of S-type asteroids. Mothé-Diniz et al. (2003) were the first to show the presence of S-types out to 3 AU in their dataset of asteroids 15 km and greater. We find that almost no S-types exist among Cybeles, and they are entirely absent beyond 3.5 AU.

Despite the bias toward discovering S-types (they reflect five times more light than C-, D-, and P- type bodies of the same size) and their abundance in the main belt, we find no convincing evidence for S-type asteroids among Hildas and Trojans. Ten asteroids among Hildas and Trojans have at least one SDSS measurement classified as S-type. Half of those objects have another observation that does not suggest an S-like composition (the second observation is typically classified D). Visual inspection suggests the quality of the data for two of them are poor. Only one object has an albedo measurement, but the low value of 0.07 is very unlikely to represent an S-type composition. One object among each of the Hildas and Trojans remains. While we cannot rule out these objects, given the other mis-categorized data and our caution against interpreting single objects, we do not find any convincing evidence from this dataset of S-types among Hildas or Trojans (or any other high albedo classes). We reach conclusions similar to the many authors who have investigated the compositions of these regions (Emery and Brown, 2003; Emery and Brown, 2004; Emery et al., 2011; Fornasier et al., 2004; Fornasier et al., 2007; Yang and Jewitt, 2007, 2011; Roig et al., 2008; Gil-Hutton and Brunini, 2008; Grav et al., 2011, 2012a,b). The wider range of albedos found among the smallest Trojans (Fernández et al., 2009; Grav et al., 2012b) which are not well-sampled in this work should prompt further follow up investigation of these targets to determine their taxonomic class. While it is possible this albedo difference with size is due to the younger age of the smaller bodies (Fernández et al., 2009), finding a wider variety of classes would prove interesting in the context of current dynamical theories such as by Morbidelli et al. (2005).

7.6. Evidence for D-types in the inner belt

We find evidence for D-types in the inner and mid belt from SDSS colors. The potential presence of D-types was also seen by Carvano et al. (2011). Here we take a scrutinizing look at the SDSS data to be certain the data are reliable.

While D-types typically have a low albedo, Bus–DeMeo D-types have been measured to have albedos as high as 0.12 (Bus and Tholen D-types have maximum albedos of 0.25). We compare the median albedo of D-types in the inner, middle, and outer belt. For samples of 35, 81 and 108 we find medians of 0.13, 0.13, and 0.08. The median albedos in the inner and middle belt suggest that there is more contamination from other asteroid classes, however, there is still a large portion of the sample with low albedos. Next we inspect the data for all SDSS D-types in the inner belt including those without albedos. We find that 9 out of the 65 objects were observed more than once and that they all remain consistent with

a D classification, all objects observed twice were twice classified as D, objects with more observations were classified as D for at least half the observations. Additionally, we check if any of the 65 D-types are members of families. We find two objects associated with the Nysa-Polana family. Because there are many C- and X-types in that family it could indicate those two objects were misclassified, however, they represent a small fraction of our sample. Because many these objects have low albedos, are not associated with C- or X-type families and have been observed multiple times and remain consistent with the D class, we have confidence in the existence of D-types in the inner belt. The orbital elements of inner belt D-types are scattered; we find no clustering of objects. The presence of D-type asteroids in the inner belt might not be entirely consistent with the influx of primitive material from migration in the Nice model. Levison et al. (2009) find that D-type and P-type material do not come closer than 2.6 AU in their model, however, their work focused on bodies with diameters greater than 40 km.

8. Overall view

We find a total mass of the main belt of 2.7×10^{21} kg which is in excellent agreement with the estimate by Kuchynka et al. (2013) of 3.0×10^{21} kg. The main belt's most massive classes are C, B, P, V and S in decreasing order (all B-types come from the spectroscopic sample, not the SDSS sample, see Section 6.1). The total mass of each taxonomic class and respective percentage of the total main belt mass is listed in Table 5. The overall mass distribution is heavily skewed by the four most massive asteroids, (1) Ceres, (2) Pallas, (4) Vesta and (10) Hygiea, together accounting for more than half of the mass of the entire main belt. Ceres, Pallas, Vesta, Hygiea are roughly 35%, 10%, 8%, and 3% respectively of the mass of the main belt (based on the total mass from this work). If we remove the four most massive bodies as shown in Table 5, the most massive classes are then C, P, S, B and M in decreasing order. The mass of the C class is six times the mass of the S class, and with Ceres and Hygiea removed, the S-types are about 1/3 and C-types/3 of their combined mass.

The distribution of each class by total mass percentage in each zone of the main belt is shown in Table 6. As we expect, E-types dominate the Hungaria region both by mass percentage and also in total number of objects, and C and S-types are the next most abundant by mass in the Hungaria region. Most of the mass of the inner belt is in Vesta, and S-types account for 4 times more mass in the inner belt than C-types (~20 and ~5% of the total mass, respectively). In the middle belt Ceres and Pallas once again make up the majority of the mass. When excluding these two bodies, C-types and S-types each make up ~30% of the mass of the middle

Table 5

Total mass of each taxonomic type. We present the total mass and fractional mass of each type. The last column is the percentage with the four most massive asteroids (Ceres, Pallas, Vesta, and Hygiea) removed. The 5 most massive classes are in bold. While we list the values two 2 decimal places as the mathematical result we do not claim accuracy to that level.

Class	Mass (kg)	Fraction (%)	Largest removed (%)
A	9.93×10^{18}	0.37	0.37
B	3.00×10^{20}	11.10	3.55
C	1.42×10^{21}	52.53	14.41
D	5.50×10^{19}	2.03	2.03
K	2.56×10^{19}	0.95	0.95
L	1.83×10^{19}	0.68	0.68
S	2.27×10^{20}	8.41	8.41
V	2.59×10^{20}	9.59	0.01
E	1.46×10^{18}	0.05	0.05
M	8.82×10^{19}	3.26	3.26
P	2.98×10^{20}	11.02	11.02
Total	2.70×10^{21}	100	45

Table 6

Percentage of mass distributed through each zone. The total for each zone summed over all classes equals 100%. In some zones there were very few (<20) objects of a certain class. We note these here to be aware of possible uncertainties do to small number statistics: A-types in all zones, B-types in Hildas, C-types in Trojans, D-, K-, L-, and S-types in Cybeles, V-types in the middle and outer belt, and X-types in Trojans.

Zone	A	B	C	D	K	L	S	V	E	M	P	Total
Hungaria	7	0	21	1	5	7	9	0	50	0	0	100
Inner	0	<1	6	<1	1	1	21	69	<1	1	<1	100
Middle	<1	15	70	<1	1	1	8	0	<1	1	4	100
Outer	<1	13	52	1	2	1	5	0	<1	10	15	100
Cybele	0	<1	13	2	<1	<1	1	0	0	<1	84	100
Hilda	0	0	14	15	0	<1	0	0	0	1	71	100
Trojan	0	0	2	67	0	<1	0	0	0	4	26	100

Table 7

Percentage of mass distribution for each class in each zone. The percentage for each class summed over all zones equals 100%.

Zone	A	B	C	D	K	L	S	V	E	M	P
Hungaria	<1	0	0	0	<1	<1	0	0	3	0	0
Inner	<1	<1	1	1	10	25	35	100	44	6	<1
Middle	94	74	74	2	37	47	51	0	45	23	21
Outer	6	26	22	8	53	27	14	0	8	67	30
Cybele	0	<1	1	5	<1	1	<1	0	0	<1	36
Hilda	0	0	<1	10	0	<1	0	0	0	<1	8
Trojan	0	0	<1	74	0	<1	0	0	0	3	5
Total	100	100	100	100	100	100	100	100	100	100	100

belt, P-types ~20% and B- and M-types ~5%. The outer belt is heavily weighted toward C-types including or excluding the most massive body, (10) Hygiea. A shift to an abundance of P-types occurs in the Cybeles. Both the Cybeles and Hildas are predominantly P-type by mass. The majority of Trojans are D-type asteroids. Based on these findings, we can confirm and recreate the general trend of E, S, C, P, and D-type asteroids with increasing distance from the Sun as established by [Gradie and Tedesco \(1982\)](#) and [Gradie et al. \(1989\)](#).

Previous work reports the distribution as relative frequency in each semi-major axis bin. Thus the results reported so far can be compared to previous work, however, previous fractions were reported by number and not by mass. This view ignores the relative importance of each semi-major axis zone. The results must be weighted according to how much mass each region holds. The Hungarias only contain about 0.02% of the mass of the inner belt for example. The total mass of each region increases moving outward, peaks in the middle belt and decreases thereafter, though the Trojans hold more mass than the Hildas. Excluding the 4 most massive objects the total mass peaks in the outer belt.

A relative weighting by mass allows us to more accurately see how each class is distributed across the belt (see [Table 7](#)). For example there have been differing views about S-type abundances. S-types are typically thought of being most abundant in the inner belt where their relative frequency is greatest ([Gradie et al., 1989](#); [Bus and Binzel, 2002a](#)). [Mothé-Diniz et al. \(2003\)](#) find S-types distributed evenly across the main belt. We report that one third of the mass of all S-types is in the inner belt, one half is actually in the middle, belt and ~15% of S-type mass is in the outer belt.

Another example of the importance of relative weighting is the E-types. E-types are typically associated with the Hungaria region. The numerous E-types ([Tholen, 1984](#); [Clark et al., 2004](#)) found in this region are thought to be a part of the Hungaria asteroid family ([Warner et al., 2009](#); [Milani et al., 2010](#)), the largest being (434) Hungaria after which this region is named. However, Hungarias only account for 3% of the mass of E-types. Nearly 90% of the mass of E-types resides in the inner and middle belt split among a few large asteroids.

While the large majority of V-type mass is contained within Vesta (although one must keep in mind this is a differentiated body

and thus has differing composition as a function of distance from the center, only the surface layer is V-type), aside from Vesta nearly 20% of the mass of V-types is in the outer belt, due to (1459) Magnya. A careful study of the distribution of V-types across the belt with follow-up observations of SDSS candidates was performed by ([Moskovitz et al., 2008](#)).

Most of the mass of C- and B-types lies in the middle belt, however, if Ceres and Pallas are excluded, the majority lies in the outer belt. While most of the mass of C-types is in the middle and outer belt, the inner belt and Cybeles contain roughly the same amount of C-type material (1%). P-types are actually relatively evenly distributed throughout the middle, outer, and Cybele regions and the Hildas and Trojans only account for a small mass (<15%) although this number is biased because discoveries among Hildas and Trojans are incomplete.

[Tables 5–7](#) provide the total amount of material for each asteroid class present in the inner Solar System and how they are distributed. The composition of each body was set at an early stage of the Solar System formation when the asteroids accreted. The subsequent dynamical history of the Solar System may have shifted their positions and greatly reduced their numbers (e.g., [Gomes et al., 2005](#); [Morbidelli et al., 2005](#); [Tsiganis et al., 2005](#); [Minton et al., 2009](#)). The masses listed in [Table 5](#) can be directly compared to the output of numerical simulations using assumptions on the original formation location of each class.

9. Conclusion

In this work we present the bias-corrected taxonomic distribution of asteroids down to 5 km. We present a method to connect the broad-band photometry of the Sloan Digital Sky Survey to previous asteroid taxonomies, based on spectra with high spectral resolution and similar wavelength range. Such a method could be applied to other multi-filter surveys. We then present a bias-correction method relevant to large datasets whereby we select the least-biased subset to account for regions and sizes adequately sampled by the SDSS survey and include a correction for discovery incompleteness of the MPC at the smallest sizes. The color-based taxonomy and bias-correction is used to study the distribution of

material in the asteroid belt for the first time according to a variety of new parameters (surface area, volume, and mass) rather than number to more accurately represent the total material. These quantities add a new perspective that was previously unachievable by studying solely the distribution by number. For instance, the E-types, are often described as unique to the Hungarias yet 90% of E-type mass is in the main belt. Additionally, the primary residence of S-types is typically thought to be the inner belt, yet we find half the mass of S-types is in the middle belt.

We confirm many trends seen in previous works with S- and V-type asteroids accounting for most of the inner belt, and C-, P-, and D-type asteroids dominating the outer belt to the Trojans. We find this view of the compositional distribution of the largest bodies determined in the early 1980s is still robust. We confirm the presence of S-types in the outer parts of the main belt as seen by Mothé-Diniz et al. (2003) as well as the scarcity of S-type among Hildas and Trojans noted by many authors. We find evidence for numerous D-types in the inner belt that were previously expected to be nonexistent in that region, and in possible contradiction with the dynamical models of implementation of trans-neptunian objects in the outer belt during planetary migration.

The main belt's most massive classes are C, B, P, V and S in decreasing order. Excluding the four most massive asteroids, (1) Ceres, (2) Pallas, (4) Vesta and (10) Hygiea that heavily skew the values, primitive material (C-, P-types) account for more than half main-belt and Trojan asteroids, most of the remaining mass being in the S-types. All the other classes are minor contributors to the material between Mars and Jupiter.

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